

134 Quiz 9 Solutions

QUESTION 1

The following series is a telescoping series.

$$\sum_{n=3}^{\infty} \left(\frac{n}{n-2} - \frac{n+2}{n} \right)$$

- Find an expression for the n -th partial sum S_n . (Do not use sigma notation or "... " in your answer here. Your answer should have a finite number of terms.)
- Using your answer to part (a.), evaluate $\lim_{n \rightarrow \infty} S_n$. Fully justify your answer.
- What conclusion can you make about the original series?
 - Does the series converge or diverge? Or, is there not enough information?
 - If the series converges, what is its sum? Or, is there not enough information?

$$\begin{aligned}
 a) \quad S_n &= a_3 + a_4 + \dots + a_n \\
 &= \left(\frac{3}{1} - \frac{5}{3} \right) + \left(\frac{4}{2} - \frac{6}{4} \right) + \left(\frac{5}{3} - \frac{7}{5} \right) + \left(\frac{6}{4} - \frac{8}{6} \right) \\
 &\quad + \dots + \left(\frac{n-2}{n-4} - \frac{n}{n-2} \right) + \left(\frac{n-1}{n-3} - \frac{n+1}{n-1} \right) + \left(\frac{n}{n-2} - \frac{n+2}{n} \right) \\
 &= \frac{3}{1} + \frac{4}{2} - \frac{n+1}{n-1} - \frac{n+2}{n} \\
 &= 5 - \frac{n+1}{n-1} - \frac{n+2}{n}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad \lim_{n \rightarrow \infty} S_n &= \lim_{n \rightarrow \infty} \left(5 - \frac{1 + \frac{1}{n}}{1 - \frac{1}{n}} - \frac{1 + \frac{2}{n}}{1} \right) \\
 &= 5 - \frac{1+0}{1-0} - \frac{1+0}{1} = \boxed{3}
 \end{aligned}$$

c) The series converges to 3
 (because $\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} S_n$ always)

QUESTION 2

Determine whether the given sequence is convergent or divergent. If it is convergent, find the limit.

$$a_n = \frac{\cos(n)}{n}$$

$$0 \leq |a_n| = \frac{|\cos n|}{n} \leq \frac{1}{n}$$

$$\text{Since } \lim_{n \rightarrow \infty} \frac{1}{n} = 0, \quad \lim_{n \rightarrow \infty} |a_n| = 0$$

$$\text{So } \lim_{n \rightarrow \infty} a_n = 0 \quad * \text{convergent, limit is } 0$$

QUESTION 3

Determine whether the given sequence is convergent or divergent. If it is convergent, find the limit.

$$a_n = \frac{5n^2 - 4n}{\sqrt{8n^3 + 3n^2 - 1}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{(5n^2 - 4n) \cdot \frac{1}{n^{3/2}}}{\sqrt{8n^3 + 3n^2 - 1} \cdot \frac{1}{\sqrt{n^3}}}$$

$$= \lim_{n \rightarrow \infty} \frac{5n^{1/2} - \frac{4}{n^{1/2}}}{\sqrt{8 + \frac{3}{n} - \frac{1}{n^3}}}$$

think:

$$\frac{\infty - 0}{\sqrt{8 + 0 - 0}}$$

$$= \infty \quad * \text{divergent}$$

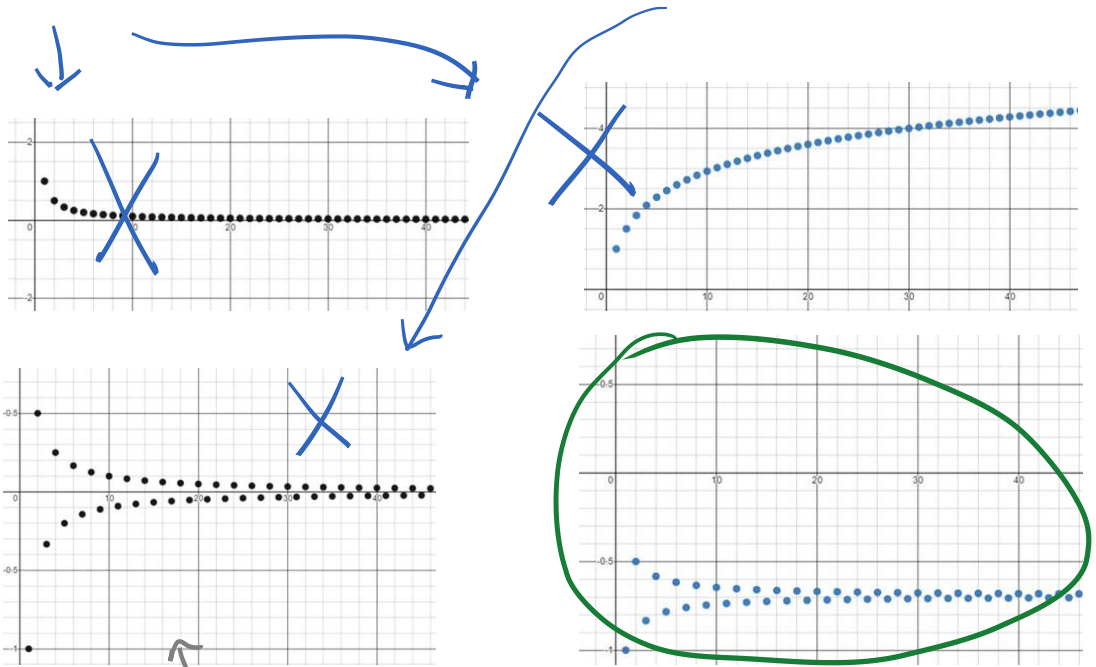
QUESTION 4

Choose the graph that shows the terms of the sequence of partial sums S_n for the given series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

$$S_1 = \frac{(-1)^1}{1} = -1$$

$$S_2 = \frac{(-1)^1}{1} + \frac{(-1)^2}{2} = -1 + \frac{1}{2} = -\frac{1}{2}$$



(this is the sequence of terms)

QUESTION 5

Determine whether the given geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} 5(8)^n(10)^{-n} = \sum_{n=1}^{\infty} 5\left(\frac{8}{10}\right)^n = \sum_{n=1}^{\infty} 5\left(\frac{4}{5}\right)^n$$

$$r = \frac{4}{5} \leftarrow |r| < 1 \text{ so convergent}$$

$$a = 5\left(\frac{4}{5}\right)^1 = 4$$

$$S = \frac{a}{1-r} = \frac{4}{1-\frac{4}{5}} = \frac{20}{5-4} = 20$$

* convergent, sum is 20

QUESTION 6

Determine whether the given geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} 6 \left(\frac{5}{3} \right)^n$$

$$r = \frac{5}{3} > 1$$

* divergent